Digital Communication Using Chaotic Pulse Generators

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Utilization of chaotic signals for covert communications remains a very promising practical application. Multiple studies indicated that the major shortcoming of recently proposed chaos-based communication schemes is their susceptibility to noise and distortions in communication channels. In this paper we discuss a new approach to communication with chaotic signals, which demonstrates good performance in the presence of channel distortions. This communication scheme is based upon chaotic signals in the form of pulse trains where intervals between the pulses are determined by chaotic dynamics of a pulse generator. The pulse train with chaotic interpulse intervals is used as a carrier. Binary information is modulated onto this carrier by the pulse position modulation method, such that each pulse is either left unchanged or delayed by a certain time, depending on whether "0" or "1" is transmitted. By synchronizing the receiver to the chaotic pulse train we can anticipate the timing of pulses corresponding to "0" and "1" and thus can decode the transmitted information. Based on the results of theoretical and experimental studies we shall discuss the basic design principles for the chaotic pulse generator, its synchronization, and the performance of the chaotic pulse communication scheme in the presence of channel noise and filtering.

I. INTRODUCTION

Noise-like signals generated by deterministic systems with chaotic dynamics have a high potential for many applications including communication. Very rich, complex and flexible structure of such chaotic signals is the result of local instability of post-transient motions in a generator of chaos. This is achieved as a result of specific features of nonlinear vector field in the phase space of the generator and not by increasing the design complexity. Even a simple nonlinear circuit with very few off-the-shelf electronic components is capable of generating a very complex set of chaotic signals. The simplicity of chaos generators and the rich structure of chaotic signals are the most attractive features of deterministic chaos that have caused a significant interest in possible utilization of chaos in communication.

Chaotic signals exhibit a broad continues spectrum and have been studied in connection with spread-spectrum applications [1]. Due to their irregular nature, they can be used to efficiently encode the information in a number of ways. Thanks to the deterministic origin of the chaotic signals two coupled chaotic systems can be synchronized to produce identical chaotic oscillations [2]. This provides the key to recovery of information that is modulated onto a chaotic carrier [3]. A number of chaos-based covert communication schemes have been suggested [4], but many of these are very sensitive to distortions, filtering, and noise [5,6]. The negative effect of filtering is primarily due to the extreme sensitivity of nonlinear systems to phase distortions. This limits the use of filtering for noise reduction in chaos-based communications. One way to avoid this difficulty is to use chaotically timed pulse sequences rather than continuous chaotic waveforms [7]. Each pulse has identical shape, but the time delay between them varies chaotically. Since the information about the state of the chaotic system is contained *entirely* in the timing between pulses, the distortions that affect the pulse shape will not significantly influence the ability of the chaotic pulse generators to synchronize and thus be utilized in communications. This proposed system is similar to other ultra-wide bandwidth impulse radios [8] that offers a very promising communication platform, especially in severe multi-path environments or where they are required to co-exist with a large number of other radio systems. Chaotically varying the spacing between narrow pulses enhances the spectral characteristics of the system by removing any periodicity from the transmitted signal. Because of the absence of characteristic frequencies, chaotically positioned pulses are difficult to observe and detect for the unauthorized user. Thus one expects that transmission based on chaotic pulse sequences can be designed to have a very low probability of intercept.

In this paper we discuss the design of a self-synchronizing chaos-based impulse communication system, and present the results of the performance analysis in the demonstration setup operating through a model channel with noise, filtering, and attenuation. We consider the case where the encoding of the information signal is based upon the alteration of time position of pulses in the chaotic train. This encoding method is called Chaotic Pulse Position Modulation [9]

II. CHAOTIC PULSE POSITION MODULATION

In this section we describe the method of Chaotic Pulse Position Modulation (CPPM) and basic elements of its hardware implementation. Consider a chaotic pulse generator which produces chaotic pulse signal

$$U(t) = \sum_{j=0}^{\infty} w(t - t_j), \tag{1}$$

where $w(t-t_j)$ represents the waveform of a pulse generated at time $t_j = t_0 + \sum_{n=0}^{j} T_n$, and T_n is the time interval between the n-th and $(n-1)$-th pulses. We assume that the sequence of the time intervals, T_i, represents iterations of a chaotic process. For simplicity we will consider the case where chaos is produced by a one-dimensional map $T_n = F(T_{n-1})$, where $F(\ )$ is a nonlinear function. Some studies on such chaotic pulse generators can be found in [7,10].

Using the Chaotic Pulse Position Modulation method the information is encoded within the chaotic pulse signal by using additional delays in the generated interpulse intervals, T_n. As a result, the generated pulse sequence is given by a new map of the form

$$T_n = F(T_{n-1}) + d + mS_n, \tag{2}$$

where S_n is the information-bearing signal. Here we will consider only the case of binary information, and therefore, S_n equals to zero or one. The parameter m characterizes the amplitude of modulation. The parameter d is a constant time delay which is needed for practical implementation of our modulation and demodulation method. The role of this parameter will be specified later. In the design of chaotic pulse generator the nonlinear function $F(\ )$, and parameters d and m are selected to guarantee chaotic behavior of the map.

The modulated chaotic pulse signal $U(t) = \sum_{j=0}^{\infty} w(t - t_0 - \sum_{n=0}^{j} T_n)$, where T_n is generated by Eq.(2) is the transmitted signal. The duration of each pulse $w(t)$ in the pulse train is assumed to be much shorter than the minimal value of the interpulse intervals, T_n. To detect information at the receiver end, the decoder is triggered by the received pulses, $U(t)$. The consecutive time intervals T_{n-1} and T_n are measured and the information signal is recovered from the chaotic iterations T_n with the formula

$$S_n = (T_n - F(T_{n-1}) - d)/m, \tag{3}$$

If the nonlinear function, $F(\ )$, and parameters d and m in the authorized receiver are the same as in the transmitter, then the encoded information, S_n, can be easily recovered. When the nonlinear functions are not matched with sufficient precision, a large decoding error results. In other words, an unauthorized receiver has no information on the spacing between the pulses in the transmitted signal, it cannot determine whether a particular received pulse was delayed, and thus whether S_n was "0" or "1".

Since the chaotic map of the decoder in the authorized receiver is matched to the map of the encoder in the corresponding transmitter, the time of the next arriving pulse can be predicted. In this case the input of the synchronized receiver can be blocked up to the moment of time when the next pulse is expected. The time intervals when the input to a particular receiver is blocked can be utilized by other users, thus providing a multiplexing strategy. Such selectivity helps to improve the performance of the system by reducing the probability of false triggering of the decoder by channel noise.

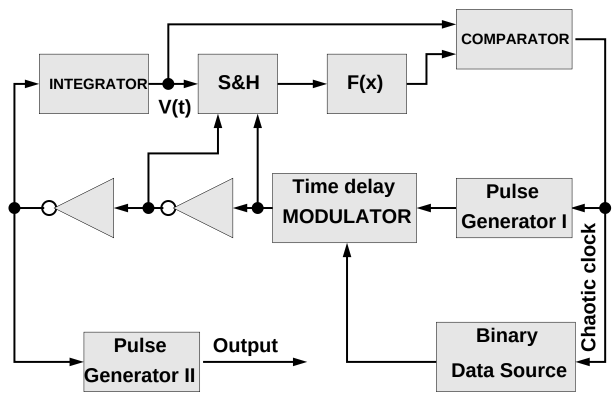

FIG. 1. Block diagram of the chaotic pulse modulator.

A. Transmitter

The implementation of the chaotic pulse modulator used in our experiments is illustrated in Fig.1. The Integrator produces a linearly increasing voltage, $V(t) = \beta^{-1}(t - t_n)$, at its output. At the Comparator this voltage is compared with the threshold voltage produced at the output of the nonlinear converter $F(x)$. The threshold level $F(V_n)$ is formed by a nonlinear conversion of voltage $V_n = V(t_n)$ which was acquired and saved from the previous iteration using sample and hold (S&H) circuits. When voltage $V(t)$ reaches this threshold level, the comparator triggers the Pulse Generator I. It happens at the moment of time $t'_{n+1} = t_n + \beta F(V_n)$. The generated pulse (Chaotic Clock Signal) causes the Data Generator to update the transmitted information bit. Depending on what the information bit S_{n+1} is being transmitted, the Delay Modulator delays the pulse produced by the Pulse Generator by the time $d + mS_{n+1}$. Therefore the delayed pulse is generated at the moment of time $t_{n+1} = t_n + \beta F(V_n) + d + mS_{n+1}$. Through the sample and hold circuit (S&H) this pulse first resets the threshold to the new iteration value of the chaotic map $V(t_{n+1}) \to F(V(t_{n+1}))$, and then resets the integrator output to zero, $V(t) = 0$. The dynamics of the threshold is determined by the shape nonlinear function $F(\ )$. The

spacing between the n-th and $(n+1)$-th pulses is proportional to the threshold value V_n, which is generated according to the map

$$T_{n+1} = \beta F(\beta^{-1} T_n) + d + m S_{n+1}, \qquad (4)$$

where $T_n = t_{n-1} - t_n$, and S_n is the binary information signal. In the experimental setup the shape of the nonlinear function was built to have the following form

$$F(x) \equiv \alpha f(x) = \begin{cases} \alpha x & \text{if } x < 5V, \\ \alpha(10V - x) & \text{if } x \geq 5V. \end{cases} \qquad (5)$$

The selection of the nonlinearity in the form of piecewise linear function helps to ensure the robust regimes of chaos generation for rather broad ranges of parameters of the chaotic pulse position modulator.

The position-modulated pulses, $w(t - t_j)$ are shaped in the Pulse Generator II. These pulses form the output signal $U(t) = \sum_{j=0}^{\infty} w(t - t_j)$, which is transmitted to the receiver.

B. Receiver

When the demodulator is synchronized to the pulse position modulator, then in order to decode a single bit of transmitted information we must determine whether a pulse from the transmitter was or was not delayed relative to its anticipated position. If an ideal synchronization is established, but the signal is corrupted by noise, the optimal detection scheme operates as follows. Integrate the signal over the pulse duration inside the windows where pulses corresponding to “1” and “0” are expected to occur. The decision on whether “1” or “0” is received is made based upon whether the integral over “1”-window is larger or smaller than that over “0”-window. Such detection scheme in the ideal case of perfect synchronization is the ideal Pulse Position Modulation (PPM) scheme. The performance of this scheme is known to be 3dB worse than the BPSK system. Although in the case of perfect synchronization this detection scheme is ideal, according to our numerical simulations, its performance quickly degrades when synchronization errors due to the channel noise are taken into account. For this reason and for the sake of design simplicity we use a different approach to detection. The demodulator scheme is illustrated in Fig.2.

In the receiver the Integrator, S&H circuits and the nonlinear function block generating the threshold values are reset or triggered by the pulse received from the transmitter rather than by the pulse from the internal feedback loop. To be more precise, they are triggered when the input signal, $U(t)$, from the channel exceeds certain input threshold. The time difference between the anticipated location of the pulse without modulation, $t'_{n+1} = t_n + \beta F(V_n)$, and the actual arrival time t_{n+1} translates into the difference between the threshold value, $F(V_n)$ generated by the nonlinear function and the voltage, $V(t_{n+1})$ at the Integrator at the moment when the input signal, $U(t)$ exceeds the input threshold. For each received pulse the difference $V(t_{n+1}) - F(V_n)$ is computed and is used for deciding whether or not the pulse was delayed. If this difference is less than the reference value $\beta(d + m/2)$, the detected data bit S_{n+1} is “0”, otherwise it is “1”.

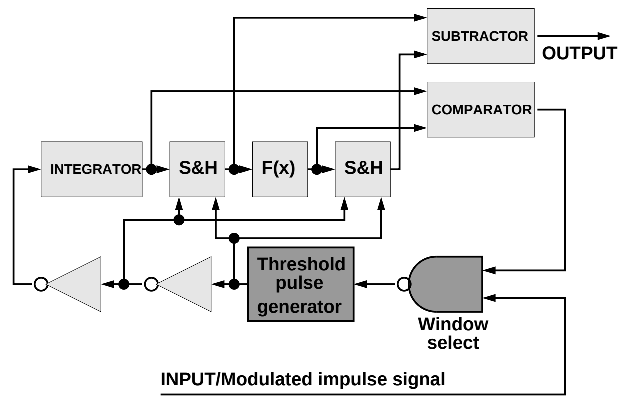

FIG. 2. Block diagram of the chaotic pulse demodulator.

Another important detail of the receiver is the Window Selection block. Once the receiver correctly observes two consecutive pulses, it can predict the earliest moment of time when it can expect to receive the next pulse. This means that we can block the input to the demodulator circuit until shortly before such a moment. This is done by the Window Select block. In the experiment this circuit opens the receiver input at the time $t'_{n+1} = t_n + \beta F(V_n)$ by Window Control pulses. The input stays open until the decoder is triggered by the first pulse received. Using such windowing greatly reduces the chance of the receiver being triggered by noise, interference or pulses belonging to other users.

C. Parameters mismatch limitations

It is known that because the synchronization-based chaotic communication schemes rely on the identity of synchronous chaotic oscillations, they are susceptible to negative effects of parameters mismatches. Here we evaluate how precisely the parameters of our modulator and demodulator have to be tuned in order to ensure errorless communication over a distortion-free channel.

Since the information detection in our case is based on the measurements of time delays, it is important that the modulator and the demodulator can maintain synchronous time reference points. The reference point in the modulator is the front edge of the Chaotic Clock Pulse. The reference point in the demodulator is the front edge of the Window Control Pulse. Ideally, if the

parameters of the modulator and the demodulator were exactly the same and the systems were synchronized, then both reference points would be always at the times $t'_{n+1} = t_n + \beta F(V_n)$, and the received pulse would be delayed by the time d for $S_{n+1} = 0$ and $d+m$ for $S_{n+1} = 1$. In this case, setting the bit separator at the delay $d+m/2$ would guarantee errorless detection in a noise-free environment.

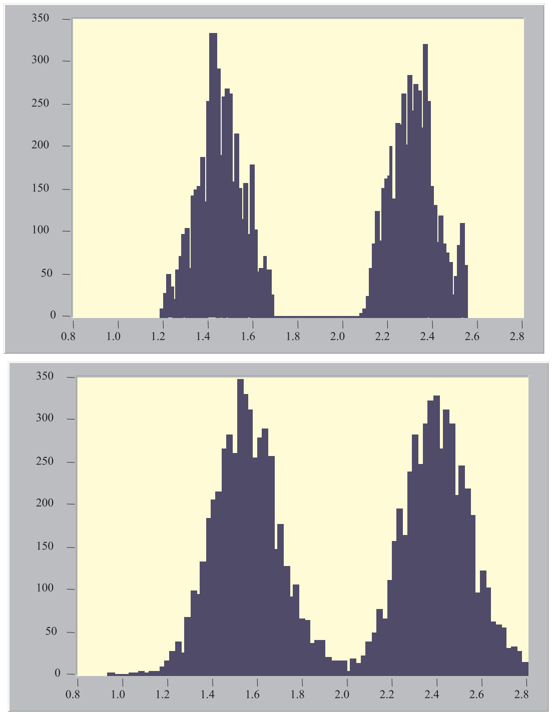

FIG. 3. Histograms of the fluctuations of the received pulse positions with respect to the receiver reference point: noise-free channel- (top) and channel with WGN $E_b/N_o \sim 18$dB - (bottom).

In an analog implementation of a chaotic pulse position modulator/demodulator system, the parameters of the circuits are never exactly the same. Therefore, the time positions $t_n'^{(M)}$ and $t_n'^{(D)}$ of the reference points in the modulator and the demodulator chaotically fluctuate with respect to each other. Due to these fluctuations the position of the received pulse, $t_n = t_n'^{(M)} + d + S_n$, is shifted from the arrival time predicted in the demodulator, $t_n'^{(D)} + d + S_n$. The errors are caused by the following two factors. First, when the amplitude of fluctuations of the position shift is larger than $m/2$, some delays for "0"s and "1"s overlap and cannot be separated. Second, when the fluctuations are such that a pulse arrives before the demodulator opens the receiver input ($t_n < t_n'^{(D)}$), the demodulator skips the pulse, loses synchronization and cannot recover the information until it re-synchronizes. In our experimental setup the parameters $\beta_{M,D}$ were tuned to be as close as possible, and the nonlinear converters were built using 1% components. The fluctuations of the positions of the received pulses with respect to the Window Control pulse were studied experimentally by measuring time delay histograms. Figure 3 presents typical histograms measured for the case of noise-free channel and for the channel with noise when $E_b/N_o \sim 18$dB.

Assuming that systems were synchronized up to the $(n-1)$-st pulse in the train, the fluctuations of the separation between the reference time positions equals

$$\Delta_n \equiv t_n'^{(D)} - t_n'^{(M)} = \beta_D F_D(\beta_D^{-1} T_{n-1}) - \beta_M F_M(\beta_M^{-1} T_{n-1}), \quad (6)$$

where indices D and M stand for demodulator and modulator respectively. As it was discussed above, in order to achieve errorless detection, two conditions should be satisfied for all time intervals in the chaotic pulse train produced by the modulator. These conditions are the synchronization condition, $\{\Delta_n\}_{max} < d$, and the detection condition $\{|\Delta_n|\}_{max} < m/2$. As an example we consider the simplest case where all parameters of the systems are the same except for the mismatch of the parameter α in the nonlinear function converter, see Eq.(5). Using Eq.(5) and Eq.(6) the expression for the separation time can be rewritten in the form

$$\Delta_n = (\alpha_D - \alpha_M)\beta f(\beta^{-1} T_{n-1}). \quad (7)$$

It is easy to show that the largest possible value of the nonlinearity output $f(\ )$, which can appear in the chaotic iterations of the map, equals to 5V. Note that in the chaotic regime only positive values of $f(\ )$ are realized. Therefore, if conditions

$$\beta(\alpha_D - \alpha_M) < d/5V \quad \text{and} \quad 2\beta|\alpha_D - \alpha_M| < m/5V. \quad (8)$$

are satisfied and there is no noise in the channel, then information can be recovered from the chaotic pulse train without errors.

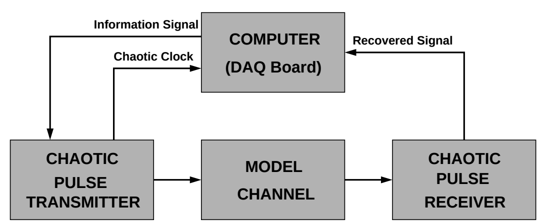

FIG. 4. Diagram of the experiment.

III. EXPERIMENTAL SETUP AND RESULTS

In our experiment (Fig.4) we used a computer with a data acquisition board as the data source, triggered by the chaotic clock from the transmitter. We also used the computer to record the pulse displacement from the

demodulator subtractor for every received pulse. This value was used to decode the information for the bit error rate analysis. The model channel circuit consisted of WGN generator and a bandpass filter with the pass band 1kHz-500kHz. The pulse duration was 500ns. The distance between the pulses varied chaotically between 12μs and 25μs. This chaotic pulse train curried the information flow with the average bit rate $\sim$60kb/sec. The amplitude of pulse position modulation, m, was 2μs. The spectra of the transmitter output, noise and the signal at the receiver are shown in Fig.5.

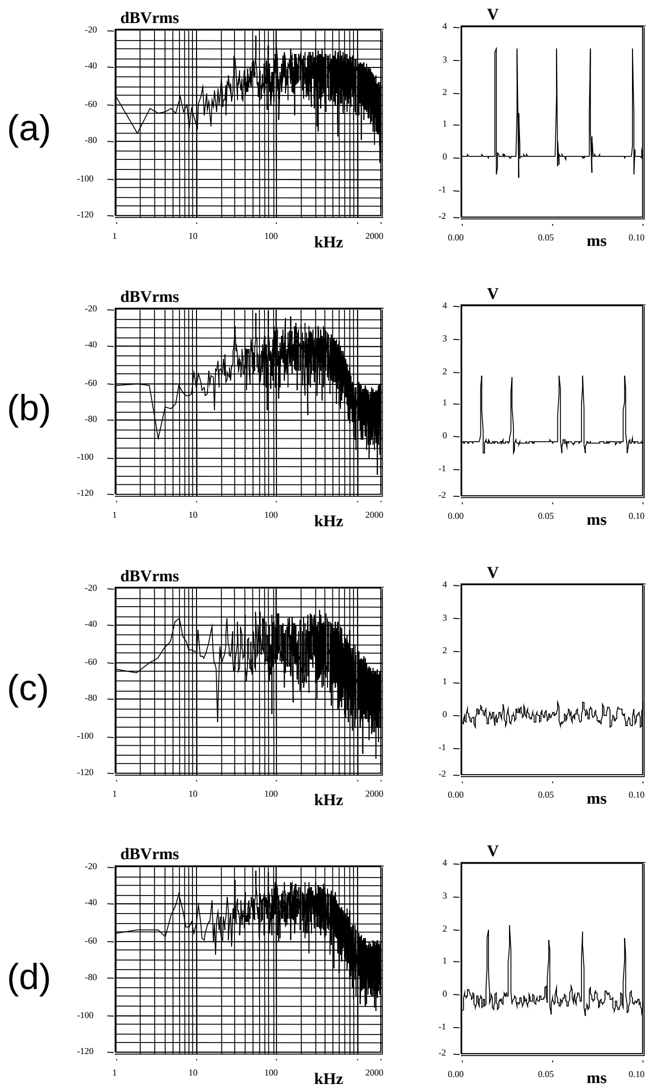

FIG. 5. Spectra and waveforms of signals in the channel: (a) – transmitter output; (b) – filtered transmitter output; (c) – filtered noise; (d) – the received signal.

We characterize the performance of our system by studying the dependence of the bit error rate on the ratio of energy per one transmitted bit to the spectral density of noise, E_b/N_0. This dependence is shown in Fig.6, where it is compared to the performance of more traditional communication schemes, BPSK, PPM, and non-coherent FSK.

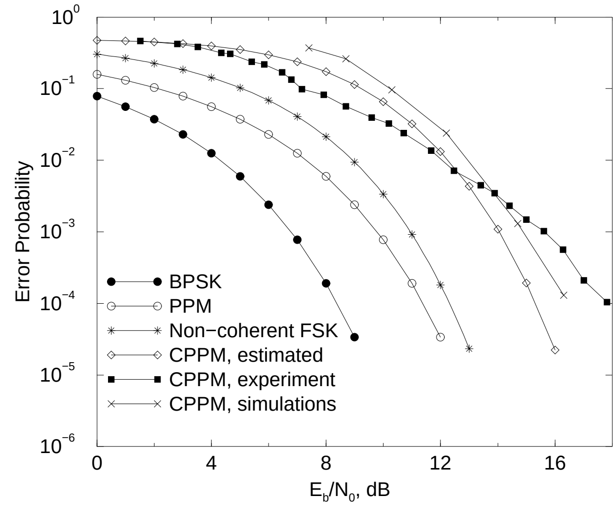

FIG. 6. Error probabilities of ideal BPSK, non-coherent FSK, and ideal PPM systems compared to the performance of the CPPM system.

We were also able to analytically estimate the performance of our system assuming perfect synchronization. The corresponding curve is also shown in figure Fig.6. At high noise levels the seemingly better performance of the experimental device compared with the analytical estimate is in part due to the crudeness of the analytical model, and in part due to the fact that at high noise level the noise distribution deviates from Gaussian. In the region of low noise the deviation of the experimental performance from the analytical estimate is probably due to the slight parameter mismatch between the transmitter and the receiver.

Discussing chaos-based communication systems, one may notice a potential disadvantage common to all such schemes. Most traditional schemes are based on periodic signals and systems where the carrier is generated by a stable system. All such systems are characterized by zero Kolmogorov-Sinai entropy h_{KS} [12]: in these systems without any input the average rate of non-redundant information generation is zero. Chaotic systems have positive h_{KS} and continuously generate information. Even in the ideal environment, in order to synchronize two chaotic systems, one must transmit an amount of information per unit time that is equal to or larger than the h_{KS} [12]. Although our detection method allows some tolerance in the synchronization precision, the need to transmit extra information to maintain the synchronization results in an additional shift of the actual CPPM performance curve relative to the case when ideal synchronization is assumed. Since the numerical and experimental curves in Fig.6 pass quite near the analytical estimate that assumes synchronization, the degradation caused by non-zero Kolmogorov-Sinai entropy does not seem to be significant.

Although CPPM performs worse than BPSK, non-coherent FSK and ideal PPM, we should emphasize that (i) this wide band system provides low probability of intercept and low probability of detection; (ii) improves the privacy adding little circuit complexity (iii) to our knowledge, this system performs exceptionally well compared to other chaos-based covert communication schemes [5]; (iv) there exist a multiplexing strategy that can be used with CPPM [13] (v) compared to other impulse systems, CPPM does not rely on a periodic clock, and thus can eliminate any trace of periodicity from the spectrum of the transmitted signal. All this makes CPPM attractive for development of chaos-based cloaked communications.

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[1] G. Mazzini, G. Setti, and R. Rovatti. Chaotic complex spreading sequences for asynchronous DS-CDMA - part I: System modeling and results (vol 44, pg 937, 1997). *IEEE Trans. Circuit. Syst. I*, 44(10):937–947, 1997.

[2] L. M. Pecora and T. L. Carroll. Synchronization in chaotic systems. *Phys. Rev. Lett.*, 64:821–824, 1990.

[3] C. W. Wu and L. O. Chua. A simple way to synchronize chaotic systems with applications to secure communication systems. *Int. J. of Bifurcation and Chaos*, 3(6):1619–1627, 1993.

[4] M. Hasler. Synchronization of chaotic systems and transmission of information. *International Journal for Bifurcation and Chaos*, 8(4):647–659, 1998.

[5] C.-C. Chen and K. Yao. Numerical evaluation of error probabilities of self-synchronized chaotic communications. *Submitted to IEEE Communications Letters*.

[6] N. F. Rulkov and L. Sh. Tsimring. Synchronization methods for communications with chaos over band-limited channel *Int. J. Circuit Theory and Applications.*, to be published.

[7] N. F. Rulkov and A. R. Volkovskii. Synchronization of pulse-coupled chaotic oscillators. In *Proceedings of the 2nd Experimental Chaos Conference*, pages 106-115, World Scientific, 1993.

[8] M.Z. Win and R.A. Scholtz. Impulse radio: how it works. *IEEE Communications Letters*, 2(2):36–8, February 1998.

[9] M. M. Sushchik *et al* Chaotic Pulse Position Modulation: a Robust Method of Communicating with Chaos *Submitted to IEEE Communications Letters*.

[10] P. A. Bernhardt Coupling of the relaxation and resonant elements in the autonomous chaotic relaxation oscillator (ACRO) *Chaos* 2: 183-199, 1992.

[11] A. R. Volkovskii and N. F. Rul'kov. Synchronous chaotic response of a nonlinear oscillator system as a principle for the detection of the information component of chaos. *Sov. Tech. Phys. Lett.*, 19:97–99, 1993.

[12] T. Stojanovski, Lj. Kocarev, and R. Harris. Applications of symbolic dynamics in chaos synchronization. *IEEE Trans. Circuit. Syst. I*, 44(10):1014–1017, 1997.

[13] H. Torikai, T. Saito, and W. Schwarz. Multiplex communication scheme based on synchronization via multiplex pulse-trains. In *Proceedings of the 1998 IEEE International Symposium on Circuits and Systems*, pages 554–557, New York, 1998. IEEE.